Improved Approximation Guarantees for the Bin Packing Algorithm MM

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The bin packing problem is to pack a sequence of items, each of size in (0,1], into bins of capacity 1 so as to minimize the number of used bins. To evaluate the performance of approximation algorithms for this problem, the asymptotic approximation ratio is often employed. Intuitively, it describes how much worse the assignment of the algorithm can become compared to the assignment of an optimal assignment for sufficiently large sequences of items. The asymptotic approximation ratio R_{ALG} of an algorithm ALG is defined as

$$R_{ALG} = \lim_{L \to \infty} \sup_{OPT(I) \geq L} \frac{ALG(I)}{OPT(I)}.$$

Here, I denotes the input item sequence, $ALG(I)$ denotes the number of bins used by algorithm ALG to pack I, and $OPT(I)$ denotes the minimum number of bins needed to pack I.

Algorithm 1 :MM**[Zhu16]**

1 Sort the item sequence I in non-increasing order.
2 Open a new bin.
3 Repeatedly pack the largest remaining item unless packing it would make the sum of item sizes in the bin exceed 1.
4 Repeatedly pack the smallest remaining item unless packing it would make the sum of item sizes in the bin exceed 1.
5 Close the bin.
6 Repeat steps 2-5 until no items remain.

Zhu[Zhu16] proposed an approximation algorithm MM, whose pseudocode is given in Algorithm 1. Its performance was analyzed by Fujiwara, Atsumi, and Yamamoto [FAY25] as $\frac{5}{4} \leq R_{MM} \leq \frac{3}{2}$. We tighten this bound by showing that $\frac{4}{3} \leq R_{MM} \leq \frac{17}{12}$, which follows from Theorems 1 and 2.

Theorem 1 *For any item sequence I,* $MM(I) \leq \frac{17}{12} \cdot OPT(I) + 3$.

Theorem 1 is shown by a case analysis with solving linear programs that formulate the assignment by MM for arbitrary item sequences.

Theorem 2 *For any integer* $m \geq 2$*, there exists an item sequence I such that* $OPT(I) = m$ *and* $MM(I) \geq \frac{4}{3} \cdot OPT(I) + 1$.

The proof of Theorem 2 is obtained by analyzing the behavior of MM for a specific item sequence.

References

[FAY25] H. Fujiwara, R. Atsumi, and H. Yamamoto. Max-min and 1-bounded space altoritbms for the bin packing problem. In *Proc. WAOA'25*, LNCS 16077, pages 127–141. Springer, 2025.

[Zhu16] Dingju Zhu. Max–min bin packing algorithm and its application in nano-particles filling. *Chaos, Solitons & Fractals*, 89:83–90, 2016.